

Everpresent Lambda II: Structural Stability

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Abstract

Ideas from causal set theory lead to a fluctuating, time dependent cosmological-constant of the right order of magnitude to match currently quoted “dark energy” values. Although this effect was predicted some time ago [1, 2], it is only more recently that a more detailed phenomenological model of a fluctuating Λ was introduced and simulated numerically [3]. In this paper we continue the investigation by studying the sensitivity of the model to some of the *ad hoc* choices made in setting it up.

1 Introduction

As explained in reference [1], a heuristic argument from causal set theory, leads one to expect fluctuations in the cosmological constant Λ which scale inversely as the square-root of the spacetime volume. (The reasoning rests on the fundamental hypothesis of spatio-temporal discreteness on one hand, and on the quantal conjugacy¹ or “uncertainty relation” between four-volume and Λ on the other hand.) Assuming that the fluctuations are centered on zero, one obtains for the current epoch the prediction $\Lambda \sim \pm 10^{-120} \rho_{Planck}$, where ρ_{Planck} is the Planck density. It is thus natural to interpret the observations pointing to an accelerating Hubble expansion as a confirmation of this prediction, since the effective energy density corresponding to the acceleration is of the same order of magnitude as the predicted fluctuations. (The sign of the fluctuations is purely random in this scenario, though with a slight bias toward positive values, as described below.)

Not only does the current value of Λ receive a natural explanation in this way, but computer simulations of a simple phenomenological model [3]

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¹This conjugacy comes out very naturally in unimodular gravity [4, 5, 6].

for the time-dependence of the fluctuations have confirmed the suggestion that a type of “tracking” behavior arises automatically, in the sense that the fluctuations would have been comparable to the ambient matter density not only now, but in the past as well (and also in the future, for as long as the expansion continues). In this way the so called “Why now?” puzzle is also resolved. (This puzzle has of course also called forth many other proposed solutions, for example [7, 8, 9]. However most of them suffer from the need for “fine tuning”.)

The model put forward in [3] derives Λ (which can be interpreted quite generally as the action S of free spacetime per unit 4-volume) from a sum of random contributions to S coming from the causal set elements within the past light cone of any given spacetime location. It contains a single phenomenological parameter which reflects both the magnitude of the individual contributions to S , and the conversion factor between spacetime volume and number of causal set elements, and which in the absence of “fine tuning” would have a value of order unity. Following [3], we will refer to this parameter, which cannot at present be obtained from first principles, as α .

How does this model produce a Λ having mean zero and fluctuations of the desired magnitude? Owing to the random signs of the individual contributions to S , their sum will vanish on average, but there will remain residual fluctuations in S of order $\alpha\sqrt{N}$. Now according to one of the fundamental assumptions of causal set theory, the volume of a spacetime region must be identified — modulo Poisson fluctuations — with the number of elements constituting that region, i.e. $V \approx N$ in natural units. Thus, the fluctuations in S produce, at a given cosmic time, a cosmological term with a typical magnitude given, as desired, by $\alpha\sqrt{N}/V \approx \alpha\sqrt{V}/V \approx \alpha/\sqrt{V}$, a time-dependent value which diminishes as the volume of the past grows.² If we crudely identify V with $(H^{-1})^4$, H being the Hubble parameter, then we can see with the help of the Friedmann equation that one should expect the magnitude of the fluctuations in Λ to track the total energy density: $\Lambda \sim \sqrt{1/V} \sim 1/\sqrt{(H^{-1})^4} \sim H^2 \sim \rho_{critical}$. And exactly this behaviour was observed in the numerical simulations of [3].

Besides the restriction to spatial homogeneity, the model of [3] contains a second *ad hoc* element of importance. To understand where it comes from, recall that the cosmological term Λ in the Einstein equation,

$$\frac{1}{\kappa}G^{ab} + \Lambda g^{ab} = T^{ab} , \quad (1)$$

has classically to be a true constant unless T^{ab} fails to be conserved (or κ fails to be a constant).³ In order to interpret the predicted time dependence

²The fluctuations are constrained by hand to be spatially homogeneous, this being probably the most serious limitation of the model as it has been developed so far.

³Here $\kappa = 8\pi G$, where G is Newton’s constant. Henceforth we take $\kappa = 1$.

we must therefore depart from the classical field equations. To that end, we first assume the universe to be homogeneous and isotropic, so that (1) reduces to a set of two ordinary differential equations in a single dependent variable, the cosmological scale factor a . For the case of a spatially flat universe, which seems to be favored by the cosmological data, these two equations are [10]

$$3H^2 - \Lambda = \rho \quad (2)$$

and

$$\frac{2\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} - \Lambda = -p, \quad (3)$$

where a dot denotes differentiation with respect to proper time τ , $H = \dot{a}/a$ is the Hubble parameter, ρ is the energy density of non-gravitational matter (including “dark matter”), and p is its pressure.

For future reference, notice that if, as is often done, we interpret $-g^{ab}\Lambda$ as an effective addition to the stress-energy T^{ab} of perfect fluid form, then (2) and (3) entail the identifications, $\rho_\Lambda = \Lambda$ and $p_\Lambda = -\Lambda$, corresponding to the “fluid equation of state”, $\rho_\Lambda = -p_\Lambda$.

Of our two equations for a , the first one or *Friedmann equation* is only of first order in time. It is therefore a constraint equation in the standard terminology, and we will refer to it in this paper as the *Hamiltonian constraint equation* or HCEq. The second equation is of second differential order, and we will refer to it as the *acceleration equation* or AccEq.

Now we wish to allow Λ to vary with time. Since doing so renders (2) and (3) incompatible, we can choose at most one of these equations as our dynamical guide. In reference [3] we let HCEq play this role, but that is not the only possibility, albeit it is a natural choice and it has the nice feature that one can interpret the resulting scheme as a *local* change to the “equation of state of Λ ”.⁴ Nevertheless, one would like to know how a different choice would affect our conclusions. What if we based the model on equation (3) rather than equation (2), or on some linear combination of the two, such as the trace of the Einstein equation (TrEq), which also offers itself as a natural choice of dynamical guide? To the extent that our phenomenological scheme proved to be robust against such modifications of its ad hoc elements, we might feel more confident that it adequately realized the underlying idea of Λ fluctuations of typical magnitude $1/\sqrt{V}$. Investigating this question of “structural stability” is the purpose of the present paper.

2 The “Mixed Equation” and Structural Stability

The nice results of [3] were obtained from HCEq (2). To what extent would they persist if we used instead an arbitrary linear combination of HCEq with

⁴That is, we can retain both Einstein equations if we treat the cosmological term as a perfect fluid, but make the altered identifications, $\rho_\Lambda = \Lambda$, $p_\Lambda = -\Lambda - \dot{\Lambda}/3H$.

AccEq?

The first thing to notice in response to this question is that not all linear combinations make sense, unfortunately, because AccEq (3) is inherently unstable, even with a non-fluctuating Λ (indeed, even for $\Lambda = 0$). In the latter setting, equation (3) is actually redundant since it is (up to a factor) just the time derivative of equation (2). Conversely, solving (3) with initial data that *exactly* satisfies (2) is equivalent to solving (2), as is well known. But if one’s initial data fails slightly to satisfy (2), the failure grows with time. This makes (3) unsuitable for numerical solution, even when $\Lambda = 0$. Still less would one expect it to be suitable for a stochastic model like ours, where random deviations from the constraint submanifold are constantly being introduced dynamically, even forgetting about the round-off and discretization errors of one’s differencing scheme.⁵

The instability in question is brought out clearly if one re-expresses AccEq in terms of the variable $D = 3H^2 - \Lambda - \rho$, which measures the degree to which the HCEq fails to be satisfied. Doing this yields for the time-dependence of D

$$\frac{\dot{D}}{3H} + D = -\frac{\dot{\Lambda}}{3H}, \quad (4)$$

or

$$a^{-3} \frac{d}{d\tau} (a^3 D) = -\frac{d\Lambda}{d\tau}, \quad (5)$$

while HCEq itself is, of course, equivalent to

$$D = 0. \quad (6)$$

For a Λ which doesn’t fluctuate, say for $\Lambda = 0$, the right hand side of equation (5) vanishes, and the time dependence of D is particularly simple:

$$a(\tau)^3 D(\tau) = \text{constant}, \quad (7)$$

or

$$D(\tau) = \left(\frac{a(\tau_0)}{a(\tau)} \right)^3 D(\tau_0). \quad (8)$$

Noting that by definition $3H^2 = \Lambda + \rho + D$, and recalling that an a^{-3} scaling is precisely that of “dust” (pressureless matter), we can interpret (8) as saying that any deviation from $D(\tau_0) = 0$ acts as a fictitious source of dust introduced at time τ_0 . In a radiation-dominated cosmos, such a term will ultimately swamp the genuine stress-energy, no matter how small it starts, since $\rho_{\text{radiation}}$ scales as a^{-4} . At that point, the relative error D/ρ reaches $O(1)$ and the behaviour of the solution becomes very different from what it would have been had the error not been introduced. If D_0

⁵The instability of the “constraint submanifold” has often plagued attempts to solve the Einstein equations numerically.

happened to be negative, the cosmos would re-contract, even though in the true solution it would have continued expanding forever. (Notice also that, for numerical purposes, the instability is much worse than (8) would make it seem. Because a varies by a factor of 10^{30} or so over the course of a simulation, it is impractical to use τ as time-parameter. Rather, something like $\log \tau$ must be used, and with respect to such a time, the instability will be exponential.)

For a time-varying Λ we find instead of (8),

$$D(\tau) = \left(\frac{a(\tau_0)}{a(\tau)} \right)^3 D(\tau_0) + \int_{\tau_0}^{\tau} d\tau' \left(\frac{a(\tau')}{a(\tau)} \right)^3 (-\dot{\Lambda}(\tau')), \quad (9)$$

which says, in effect, that a variation $d\Lambda$ introduces into the effective energy density a fictitious dust contribution of $-d\Lambda$ (as if an amount $d\Lambda$ of Λ had “turned to dust”). Although this expression is less easy to analyze than (8), it is hard to believe that the fluctuations in Λ under the integral sign could avoid exciting the instability. Indeed, simulations we performed with a fluctuating Λ exhibited an instability in this case that was as bad as or worse than that for $\Lambda = 0$.

As a check on this conclusion, we also simulated AccEq another way, which is perhaps worth reporting here since it brings out the fact that our scheme can be interpreted as a modification to the “equation of state of Λ ”, rather than as a change to the Einstein equations. In this re-interpretation, we write the Einstein equations as

$$3H^2 = \rho_{\Lambda} + \rho, \quad \frac{2\ddot{a}}{a} + H^2 = -(p_{\Lambda} + p).$$

The scheme based on (2) is then equivalent to the equation of state,

$$\rho_{\Lambda} = \Lambda, \quad p_{\Lambda} = -\Lambda - \frac{d\Lambda/d\tau}{3H}, \quad (10)$$

according to which p_{Λ} depends on both Λ and its time-derivative,⁶ while the scheme based on (3) is equivalent to the equation of state

$$p_{\Lambda} = -\Lambda, \quad \rho_{\Lambda} + \dot{\rho}_{\Lambda}/3H = \Lambda, \quad (11)$$

according to which ρ_{Λ} is not even a local (in τ) function of Λ . As expected (since the two approaches are equivalent modulo numerical errors) simulations with this scheme exhibited the same instability as did direct simulation of equation (3).

Now let us extend this stability analysis to the equation formed as an arbitrary linear combination (with coefficients ν, μ) of HCEq (2) with AccEq

⁶An equation of state like this was considered in [12]. Notice that, strictly speaking, $d\Lambda/d\tau$ is infinite because Λ is (in the continuum limit) a function of Brownian type.

(3). Expressing the resulting equation in terms of D yields the corresponding linear combination of (6) and (4), namely

$$\mu \frac{\dot{D}}{3H} + (\mu + \nu)D = 0 , \quad (12)$$

where we have again taken $\Lambda = 0$. Instead of (7), we find now

$$D = \frac{\text{constant}}{a^{3b}} , \quad (13)$$

where $b = (\mu + \nu)/\mu = 1 + \nu/\mu$. In a radiation-filled cosmos, we therefore need $3b \geq 4$, in order that the error remain small.⁷ The limiting case is thus $b = 4/3$ or $\mu = 3\nu$. Interestingly, this corresponds exactly to the trace equation, TrEq (given by $\nu = 1$, $\mu = 3$, and equivalent to $G_a^a + 4\Lambda = T_a^a$), which accordingly is “marginally unstable” in the structural sense.⁸ The trace equation (TrEq) and the Friedmann equation (HCEq) are then at the two extremes of the stable range, and one may expect that every other (stable) combination will behave in a way intermediate between these two extremes. This makes TrEq all the more interesting, supplementing its intrinsic interest in relation to conformal variations of the metric.

In the rest of this paper, we will refer to the above linear combination of AccEq with HCEq as *MixedEq*, and we will limit μ/ν to its stable range. In terms of conformal time, MixedEq is given by

$$\mu a'' = \frac{\mu - 3\nu}{2} \frac{(a')^2}{a} + \frac{a^3}{2} (\nu \rho_{\text{total}} - \mu p_{\text{total}}) + \frac{a^3}{2} (\mu + \nu) \Lambda \quad (14)$$

with

$$\rho_{\text{total}} = \rho_{\text{radiation}} + \rho_{\text{matter}} ,$$

and

$$p_{\text{total}} = \frac{\rho_{\text{radiation}}}{3} .$$

Here $\rho_{\text{radiation/matter}}$ is the energy density in radiation/pressureless matter, and a prime represents differentiation with respect to the conformal time, $\eta = \int d\tau/a$. For $\mu = 3\nu$ we obtain the TrEq written as

$$\frac{a''}{a^3} = \frac{4\Lambda + \rho_{\text{matter}}}{6} . \quad (15)$$

(Notice that ρ_{rad} drops out of TrEq, as it had to given the conformally invariant nature of radiation. For a radiation filled universe with $\rho_{\text{matter}} =$

⁷For completeness we mention that if later on the universe becomes “matter dominated”, since in that era mass density dilutes as a^{-3} , the inequality can be loosened slightly.

⁸At the other end of the stable range, HCEq betrays no hint of instability when considered on its own. But adding even a small amount of AccEq increases its differential order, whence it is not strange that such a change can destabilize it when the sign of the addition is unfavorable.

$\Lambda = 0$, (15) yields simply $da/d\eta = \text{constant}$.) We will see in the next section that equation (14) with $0 < \mu \leq 3\nu$ is indeed stable and produces results similar to those obtained in reference [3] from HCEq.

3 Results of simulations

Computer simulations of MixedEq (14) in the stable regime, $\mu \leq 3\nu$, produce results similar to those obtained in [3] from HCEq. Figure 1 displays plots

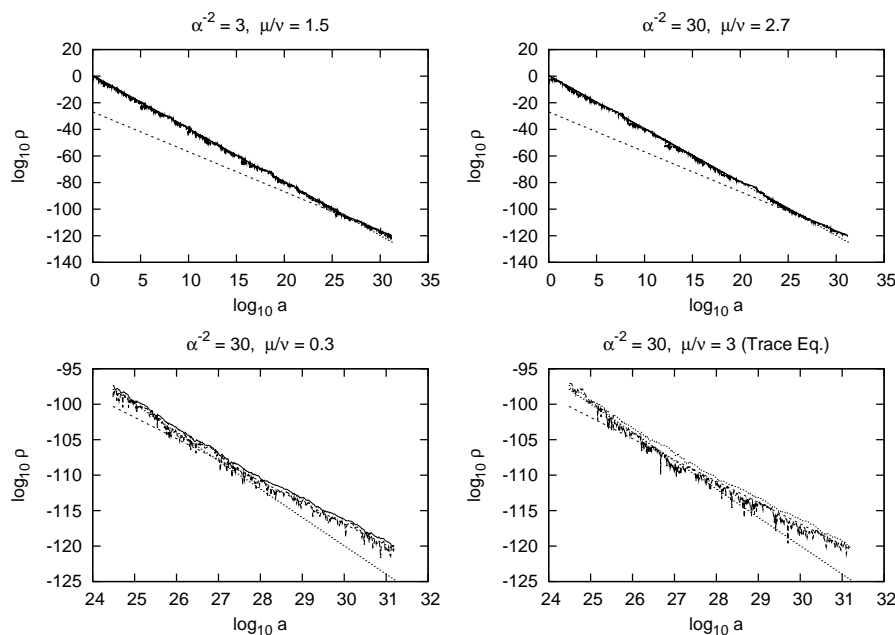


Figure 1: Tracking behaviour exhibited by $|\Lambda|$ for various values of the model-parameters μ/ν and α .

of the energy density versus the scale factor in some of these simulations, and the tracking behaviour alluded to earlier is clearly visible. The top two diagrams cover the whole period from the Planck time to the present epoch, while the bottom two diagrams focus on the transition from a^{-4} scaling (radiation domination, shown by the dotted line) to a^{-3} scaling (matter domination, shown by the dashed line) of the ambient energy density. One sees that the effective energy density in Λ (solid jittery line) switches its scaling as well and follows the total effective energy density (wiggly dotted line) modulo fluctuations.

One of the benefits of using the MixedEq (i.e., any of the stable combinations with non-vanishing μ) is the ability to accommodate larger values of the parameter α . With HCEq almost all of the simulations terminate prematurely when α is too big, the reason being that when α is large, large

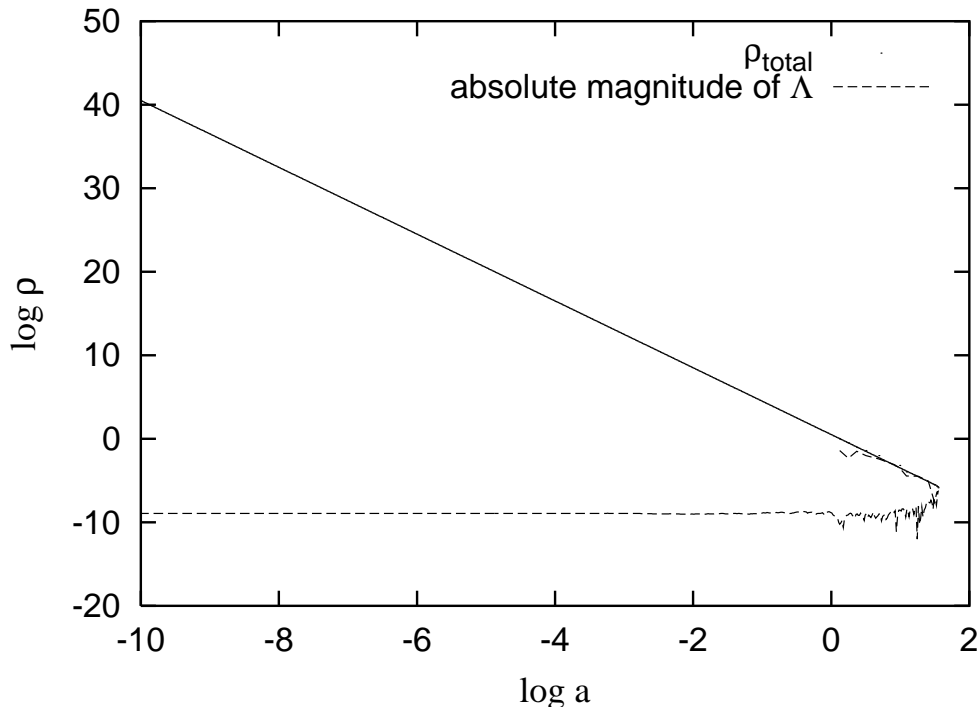


Figure 2: Recollapse is typical of evolution with MixedEq (or TrEq), and prevents the cosmos from reaching its present size when α is too large. In the example shown $\alpha = 1/5$. The energy density in matter and radiation (solid line ρ) moves from left to right as the cosmos expands, then retraces its path, heading toward the final singularity. In contrast, $|\Lambda|$ (dashed line) tracks ρ while the cosmos expands, but remains essentially frozen as it contracts.

fluctuations in Λ are bound to happen, including negative ones. If such a fluctuation overwhelms ρ in (2), it renders further evolution impossible by making H imaginary. There are intriguing suggestions of how to keep going in such a circumstance, some of which were discussed in [3], but a good understanding of their status is still lacking.⁹

On the other hand for MixedEq (14) with $0 < \mu \leq 3\nu$ a large negative fluctuation in Λ simply means that \ddot{a} becomes negative.¹⁰ This can eventually turn an expansion into a contraction (and vice versa for a positive fluctuation), but it poses no problem of principle and it lets the simulation continue. Thus we have been able to simulate as high as $\alpha^2 = 1/3$ with MixedEq, whereas $\alpha^2 > 1/50$ was not viable in [3]. The use of larger values of α has another, important benefit as it naturally results in larger final

⁹The most obvious suggestion is simply to change the sign of \dot{a} at that point, but it seems that this alone cannot cure the problem in general.

¹⁰Actually a'' becomes negative but this implies that \ddot{a} is also negative as can be seen from the equation $\ddot{a} = \frac{a''}{a^2} - \frac{a'^2}{a^3}$.

values of Ω_Λ . We will discuss this further in connection with figure 6.

Although MixedEq permits us to evolve through a turning point, thereby affording a larger latitude in the choice of α , simulations with $\alpha \gtrsim 1$ still do not reach the present value of the scale factor (or equivalently the temperature) in any significant number before re-collapsing. Thus, the problem that affected HCEq [3] shows up in another guise with MixedEq, albeit at larger values of α . The biggest α that reached the present epoch was $\sqrt{1/3} \approx 0.58$, and out of 10 million runs with this α only 25 did so. (The first graph in figure 1 shows one of these simulations). In order to explain what goes wrong, recall that the magnitude of Λ in our model decreases with the past volume (which, given our ansatz, is constant on each hypersurface of homogeneity). As long as the cosmos is expanding, the absolute magnitude of the Λ term remains comparable to the energy density in radiation (this being the relevant energy component at early times).¹¹ But when the cosmos begins to contract, the density of radiation, being proportional to a^{-4} , increases sharply. And since the past-volume keeps on accumulating, after a while the Λ fluctuations are too small to reverse the sign of \dot{a} , and the probability for the universe to recover from the collapse practically vanishes. The graph of Figure 2 illustrates one such case.

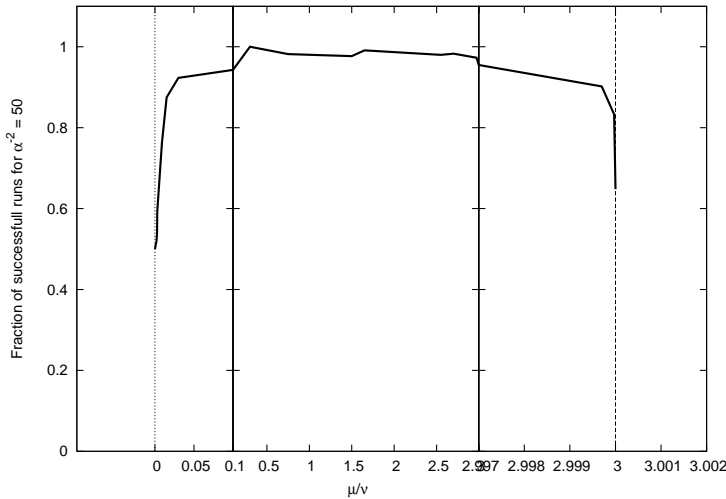


Figure 3: The fraction of runs that reach the present time, plotted as a function of μ/ν , for $\alpha^{-2} = 50$. The HCEq corresponds to $\mu/\nu = 0$, the TrEq to $\mu/\nu = 3$.

As an indicator of how our model reacts as we move away from the HCEq ($\mu = 0$) into the MixedEq ($0 < \mu < 3\nu$) and then reach the other extreme at the TrEq ($\mu = 3\nu$), we studied the fraction of times our simulations reached

¹¹This of course is the tracking behaviour that solves the “Why Now?” puzzle.

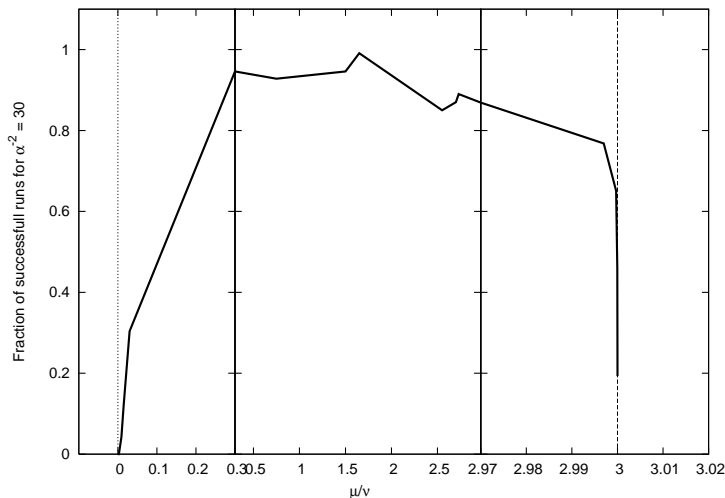


Figure 4: Same as figure 3 but for $\alpha^{-2} = 30$.

the present value of the scale-factor. For $\alpha^2 = 1/30, 1/50$, and $1/70$, we simulated HCEq, TrEq, and MixedEq with values of μ/ν as small as 0.002 (just after the HCEq) and reaching as close as 2.99998 to the TrEq ($\mu/\nu = 3$). Figures 3 and 4 plot this fraction as a function of μ/ν for $\alpha^2 = 1/50$, and $1/30$, respectively. In both plots, one observes a central “plateau” in which most realizations reach today’s scale-factor, flanked on each side by “escarpments” in which the fraction of such realizations falls precipitously as one approaches the unstable range. Close to the TrEq the dependence seems to be exponential, so in figure 5 we have expanded that part of the curve for $\alpha^2 = 1/50$ to reveal the continuous transition to $\mu/\nu = 3$.

Figure 6 shows some histograms of the final values of Ω_Λ (called Ω_Λ^{final}) for HCEq with $\alpha^2 = 1/60$ and for MixedEq with $\alpha^2 = 1/30$, and for various μ/ν values. It can be seen that the values of Ω_Λ^{final} are nicely peaked at zero and fall off on either side as expected. The first thing to notice is that Ω_Λ^{final} goes as high as 0.99. About 5 percent of the time, it lies above 0.5, and about 2 percent above 0.7. The positive values of Ω_Λ^{final} slightly outweigh the negative ones, the probable reason being that a universe which spends more of its time with positive Λ is more likely to reach the present epoch without recollapsing. In general the positive values seem to make up fifty to sixty percent of the total. One more thing that catches the eye is that the tail on the negative side is longer than on the positive side. This merely reflects the asymmetry built into the definition, $\Omega_\Lambda \equiv \rho_\Lambda/(\rho_\Lambda + \rho_m)$, in consequence of which Ω_Λ normally¹² is bounded by +1 on the positive

¹²It can happen that $\rho_\Lambda + \rho_m$ becomes negative such that $\Omega_\Lambda > 1$. However the re-contraction that accompanies a negative net effective energy density enhances the matter-

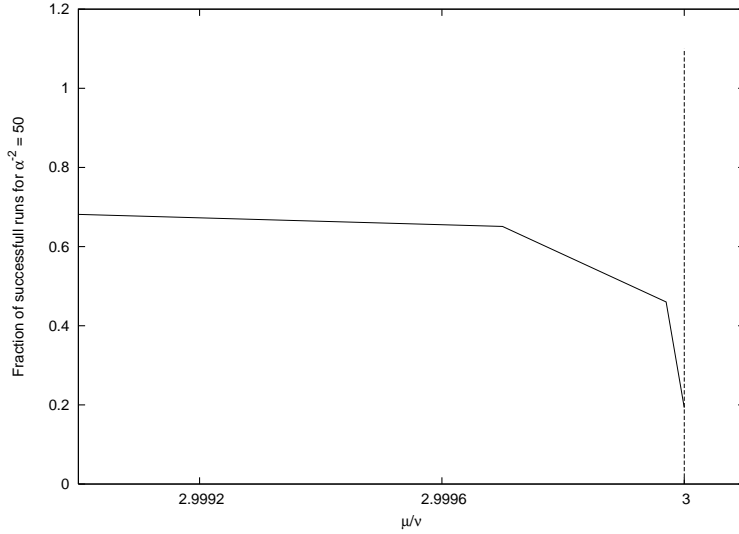


Figure 5: The end part of the curve in figure 3 magnified about twenty times.

side, while ranging to $-\infty$ on the negative side. (A few percent of the Ω_Λ fell below -1 , but we did not plot them on the histograms.)

4 Conclusions

This study is a follow-up to reference [3], where we recalled how the causal set idea leads to fluctuations in the cosmological term and showed that if the mean value of Λ is taken to be zero then these fluctuations have the potential to account for the presently observed Ω_Λ . We also raised some concerns about the model, namely:

- The nice results of [3] were obtained from the choice of equation (2) as “dynamical guide”, and it was uncertain whether (3) or a linear combination of the two equations would behave similarly.
- It was not clear how to continue the evolution when the total effective energy density became negative.
- The parameter α that governed the magnitude of the fluctuations had to be five to ten times smaller than unity, and this looked like a mild fine tuning problem.
- Rarely did the model produce a final value for Ω_Λ large enough to accord with present observations.

density, and Ω_Λ rapidly returns to its “normal” range, either for this reason or because Λ fluctuates back to a positive value.

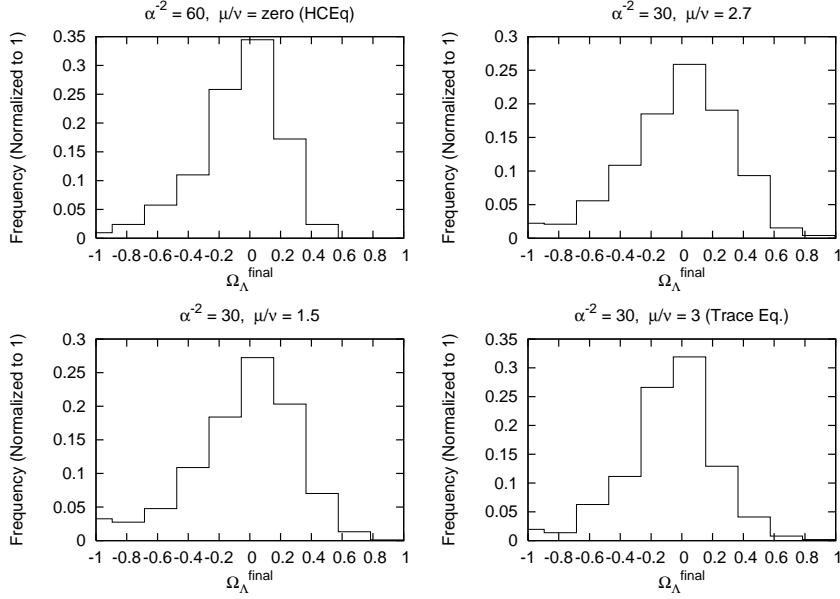


Figure 6: Histograms of the final values of Ω_Λ for $\mu/\nu = 1.5, 2.7$ and 3 ($\alpha = \sqrt{1/30}$). Also shown for comparison is a histogram for $\mu = 0$ and $\alpha = \sqrt{1/60}$.

The present work throws light on all of these issues. We take them in turn.

- It turns out that the second order equation (3), referred to in this paper as the AccEq, is inherently unstable and cannot replace HCEq (2) in the model of [3]. However a large range of convex combinations of the two equations is stable, specifically combinations of the form $\nu \times \text{HCEq} + \mu \times \text{AccEq}$ with $0 \leq \mu \leq 3\nu$. In our simulations, all of these stable combinations exhibited the same tracking behaviour as was obtained in [3] from the purely first-order HCEq ($\mu = 0$). Our question about “structural stability” is therefore answered in the affirmative.
- When the net effective energy-density of a universe (evolving under one of the stable mixtures of HCEq and AccEq) becomes sufficiently negative, an expanding universe starts to contract. In principle, it can re-expand if the net effective energy-density becomes positive again, but this happens only rarely. In any case, there is for $\mu > 0$, no problem in following an expansion and re-contraction all the way down to a singularity of infinite density.
- Whereas in [3] with the HCEq, we were only able to simulate as high as $\alpha^2 \approx 1/50$, we have been able with MixedEq to go as high as $\alpha^2 = 1/3$,

albeit $1/30$ is a more realistic upper limit if the universe is to reach its present size with an appreciable probability.

- Since with MixedEq we can handle larger values of α (with their concomitant larger fluctuations), we can end up with larger final values of Ω_Λ . For $\alpha \sim 1/30$ we can account for the present observational value of Ω_Λ in about 2 percent of the simulations.

It is important to realize, however, that our variable Ω_Λ cannot be compared directly with observational parameters quoted in the literature which assume that Λ is constant (or has an “equation of state” with constant $w = p_\Lambda/\rho_\Lambda$). When instead, Λ is fluctuating, the comparison with observation must be derived anew. To that end, one might for example use our simulations to construct fictitious data sets of supernova luminosity and redshift, one for each run of the simulations. One would then feed these data sets into one of the algorithms people have used to obtain the quoted values of Λ and w , and one would ask how often the resulting values came near to the quoted ones.

Can the accelerating Hubble expansion be traced to a fluctuating Λ and can such fluctuations be understood as a nonlocal and quantal residue of an underlying spatio-temporal discreteness?

This long-standing idea has still to be embodied in a fully fledged phenomenological model, but an encouraging start was made in [3]. The model of [3] involved some ad hoc choices, however, together with an artificial restriction to spatial homogeneity. In this paper we have not dealt with the latter shortcoming¹³ but have concentrated on the ambiguity inherent in the manner in which [3] implemented the idea of a varying Λ . So far, the evidence is that none of the qualitative features of the model depend on how that ambiguity is resolved, and overall our results sustain the picture developed in [3], according to which the fluctuating Λ tends to remain, throughout the phase of cosmic expansion, in rough equilibrium with the ambient matter density.

There remains, though, a kind of tension between the magnitude of the fluctuations (as reflected in the parameter α) and the continuation of the expansion. If α is too big, the negative fluctuations will tend to terminate the expansion; if it is too small, the positive fluctuations will be unable to account for the current value of Λ . If one chooses α appropriately, the two competing effects can be balanced¹⁴ fairly well, but some discrepancy always seems to remain. Indeed, it is likely a generic feature of the models based on MixedEq that for any choice of α , the cosmos given long enough, will eventually recollapse due to a negative fluctuation.

¹³The consequent risk to the model was pointed out in [3] and emphasized further in [18].

¹⁴Order of magnitude estimates suggest that this balancing is possible only in $3 + 1$ dimensions, as pointed out also in [18].

If this is so, we hope we may be excused for speculating further that cycles of expansion and contraction would succeed each other indefinitely, their characteristic lifetime depending on the value of α as it emerges at the start of each new cycle. In this way, the “Tolman-Boltzmann” scenario of [19] would have acquired a possible basis in quantum gravity. Although that scenario emerged from a non-quantum dynamics for causal sets, namely that of *classical sequential growth*, it illustrated how the most striking features of our universe — its approximate spatial homogeneity and isotropy — might have emerged dynamically over the course of repeated expansions and collapses. There the collapses occurred via rare statistical fluctuations following exponentially long periods of stasis. Here, the collapses would be more dynamical in character, initiated by *quantal* fluctuations in Λ , or in other words fluctuations in the form of the gravitational field equations.

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